

Scalar Trapping and Saxion Cosmology

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Abstract

We study in detail the dynamics of a scalar field in thermal bath with symmetry breaking potential. In particular, we focus on the process of trapping of a scalar field at an enhanced symmetry point through the thermal/non-thermal particle production, taking into account the interactions of produced particles with the standard model particles. As an explicit example, we revisit the saxion dynamics with an initial amplitude much larger than the Peccei-Quinn scale and show that the saxion trapping phenomenon happens for the most cases and it often leads to thermal inflation. We also study the saxion dynamics after thermal inflation, and it is shown that thermal dissipation effect on the saxion can relax the axion overproduction problem from the saxion decay.

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1 Introduction

After the discovery of the Higgs boson at the LHC [1], the concept of spontaneous symmetry breaking induced by the vacuum expectation value (VEV) of a scalar field becomes more important. Actually, apart from the electroweak symmetry breaking, the Universe may have experienced several phase transitions such as the Peccei-Quinn (PQ) phase transition [2], grand unified theory (GUT) phase transition and so on.

Most of these phase transitions are thought to be associated with VEVs corresponding scalar fields, ϕ . While the concept is simple, the dynamics of the scalar fields leading to the symmetry breaking might be rather complicated and may have significant cosmological

implications. The usual argument is as follows [3]. At high temperature, ϕ sits at the origin $\phi = 0$ due to thermal effective potential since it is an enhanced symmetry point. As the temperature decreases, thermal effects become less significant, and finally ϕ relaxes to the minimum. However, the assumption that ϕ starts at the point $\phi = 0$ may not always be justified, because ϕ can have very large initial value during and just after inflation. Then we need to study dynamics of a scalar in order to see whether or not the symmetry is restored (i.e., $\phi = 0$) in the early Universe.

If the initial amplitude of the scalar field is displaced from the minimum of the potential, the subsequent dynamics of the scalar field is non-trivial. With such an initial condition, the scalar field oscillates and dissipates its energy via the interaction with background. If the effect of the cosmic expansion is negligible, the scalar field eventually relaxes to an equilibrium state, which is determined by the initial energy (and other conserved quantities). In such a case, the scalar field is expected to be trapped at the enhanced symmetry point if the temperature of the equilibrium state is higher than the critical temperature for the phase transition. On the contrary, if the cosmic expansion alone efficiently reduces the energy of oscillating scalar field, the scalar field cannot experience the restoration of the symmetry and results in the symmetry breaking vacuum. The actual situation lies somewhere between these two extreme cases. Therefore, to see whether the scalar field experiences the trapping or not, we have to study in detail the dynamics of scalar field and compare the time scale of dynamics with that of cosmic expansion. On this respect, the phenomenon called non-thermal phase transition [4] and moduli trapping [5] was studied previously.

The complete analysis of dynamics of scalar field in a concrete setup including interactions with standard model (SM) sector in thermal environment has been lacking. In a realistic setup, such a scalar field often interacts with gauge-charged matters and the background is filled with thermal plasma after inflation. The primary purpose of this paper is to analyze the scalar dynamics with symmetry breaking potential by taking into account all possible relevant effects: thermal correction to the scalar potential, dissipative effects on the scalar field, non-perturbative particle production and the resulting modification on the scalar potential.

To make our discussion concrete, we will pay particular attention to the dynamics of saxion in a class of supersymmetric (SUSY) axion model. We will show that, even if the initial amplitude of the saxion is much larger than the PQ scale, the zero-mode of the saxion field is eventually trapped at the symmetry enhanced point in large parameter space; this is due to the modification of the saxion potential and the dissipation of the oscillation. Thus, the thermal inflation [6, 7, 8] is quite often caused by the saxion. We also study the scalar dynamics after phase transition. We find the situation where thermal dissipation into SM plasma efficiently reduces the saxion energy density without axion overproduction.

This paper is organized as follows. In Sec. 2, we introduce a basic setup and review thermal effects and particle production processes as preparations for the analyses in the following sections. In Sec. 3, we study the saxion dynamics in detail in the cases of both

small and large mixing between PQ and SM quarks. We will study in detail whether the saxion is trapped at the origin or not by taking account for thermal and non-thermal effects in the following sections (Sec. 3.2 and Sec. 3.3). In Sec. 4, we will discuss the saxion dynamics after thermal inflation and its cosmological consequence. Sec. 5 is devoted to the conclusion and discussion.

2 Preliminaries

2.1 Overview

Here, we consider the evolution of a complex scalar field which is responsible for the spontaneous breaking of a global $U(1)$ symmetry; the (zero-temperature) potential of ϕ has a minimum at $|\phi| = v \neq 0$. Above this scale, the potential of ϕ , which is lifted by the SUSY breaking effect, is assumed to be quadratic:

$$V \simeq m^2 |\phi|^2 \quad \text{for } |\phi| \gg v. \quad (2.1)$$

In addition, the scalar ϕ is assumed to have interactions with scalar \tilde{Q} and fermion Q as

$$\mathcal{L} = \lambda_s^2 |\phi|^2 |\tilde{Q}|^2 + (\lambda_f \phi \bar{Q} Q + \text{h.c.}), \quad (2.2)$$

with coupling constants λ_s and λ_f . \tilde{Q} and Q are assumed to have SM gauge interactions. Having a SUSY model in mind, we call Q and \tilde{Q} as quark and squark, respectively, hereafter.

How does the phase transition proceed? As described in the Introduction, the usual arguments are as follows. At high temperature, ϕ sits at the origin $\phi = 0$ due to thermal effective potential. In our case, it may be caused by the interaction with \tilde{Q} and Q . As the temperature decreases, thermal effects become less significant, and finally ϕ relaxes to the minimum $|\phi| = v$.

However, it is non-trivial whether ϕ actually starts at the point $\phi = 0$. Let us assume that initially, just after inflation, ϕ is placed far from the origin: $|\phi| \gg v$. This assumption is quite natural if $m \ll H_{\text{inf}}$, where H_{inf} denotes the Hubble scale during inflation. An immediate consequence is that since \tilde{Q} and Q get large masses from the VEV of ϕ , they may decouple from thermal bath even after the reheating, depending on the reheating temperature. Thus we need to investigate the dynamics of ϕ in order to see whether ϕ will be eventually trapped at $\phi = 0$ or not.

- If $|\phi|$ is large enough, \tilde{Q} and Q decouple from thermal bath. Still, however, ϕ feels thermal effects through the so-called thermal logarithmic potential [9], which may tend to stabilize ϕ toward the origin.
- The dissipation is essential to effectively reduce the coherent oscillation energy density [10, 9, 11, 12, 13, 14, 15]. In particular, we must take into account thermal dissipation effects on the scalar in order to see ϕ truly relaxes to the origin.

- In the limit of low reheating temperature, such thermal effects would be inefficient. Even in such a case, because of the large amplitude of ϕ , non-perturbative particle production events happen at each oscillation of ϕ . Then, the finite number density of \tilde{Q} and Q modifies the effective potential of ϕ which also tends to stabilize ϕ at the origin.
- The produced \tilde{Q} and Q decay or annihilate depending on their interactions to the SM particles. If Q has the same gauge quantum numbers as of right-handed down-type quarks, for example, we may obtain the following mixing term

$$\mathcal{L}_{\text{mix}} = \kappa Q q_L H + \text{h.c.}, \quad (2.3)$$

where q_L and H are SM left-handed quark and Higgs boson, respectively. With the above interaction, Q soon decays and generates thermal plasma. Even without such mixings, annihilation processes into SM particle also reduce \tilde{Q} and Q , which would also generate thermal plasma. Through these processes, ϕ coherent oscillation effectively lose its energy by producing thermal plasma, which may significantly affect the scalar dynamics.

Thus the actual dynamics of the scalar ϕ would be complicated due to the combinations of these effects. We will analyze the scalar dynamics in Sec. 3 with an explicit example of the saxion in SUSY axion model, but we emphasize that the analysis can be applied to general class of scalar fields with symmetry breaking potential. In the rest of this section, we briefly summarize the thermal effects and particle production processes.

2.2 Thermal and non-perturbative effects on scalar dynamics

When the primordial inflation ends, the scalar field ϕ may be displaced far from the true vacuum $|\phi| = v$. Eventually, it begins to relax towards the equilibrium state which is not necessarily identical to the true vacuum because there is a background thermal plasma. Even with this initial condition, which is far from the origin of potential, the scalar field may be trapped at the origin due to thermal effects since the (s)quarks become massless at the origin of scalar potential. In addition, even if thermal effects are not efficient, the scalar field can be trapped by the explosive production of squarks via the parametric resonance [16, 17].

Now let us summarize the effect of background thermal plasma at the beginning of scalar coherent oscillation and also the conditions for non-perturbative particle production in the presence of thermal plasma [15, 18]. We will discuss whether the saxion is trapped at the origin or not by taking account for thermal and non-thermal effects in the following sections (Sec. 3.2 and Sec. 3.3).

2.2.1 Thermal effects at the beginning of oscillation

Since the (s)quarks are in the thermal plasma if $\lambda|\phi| \lesssim T$, there might sink at the origin of free energy. At the one-loop level, the free energy is given by [19]

$$V_{1\text{-loop}} = \sum_i (-)^{|i|} \frac{T^4}{\pi^2} \int_0^\infty dz z^2 \ln \left[1 - (-)^{|i|} e^{-\sqrt{z^2 + M_i^2(|\phi|)}/T} \right] \quad (2.4)$$

where i represents the species (normalized by one complex scalar or one chiral fermion), $|i|$ denotes the statistical property: $|i| = 0, 1$ for boson and fermion respectively, and M_i is the mass of i that depends on $|\phi|$. If the mass is smaller than the temperature $M_i \ll T$, then one can find that this term encodes the so-called “*thermal mass*” term:

$$V_{1\text{-loop}} \supset \sum_{i \in \text{Boson}} \frac{1}{12} T^2 M_i^2(|\phi|) + \sum_{i \in \text{Fermion}} \frac{1}{24} T^2 M_i^2(|\phi|). \quad (2.5)$$

On the other hand, if $M_i \gg T$, this one-loop contribution rapidly vanishes. Instead, higher loop effects dominate the free energy, which arise from the threshold correction to the gauge coupling constant g at T . That is, using the fact that the free energy of hot plasma has a contribution proportional to $g^2(T)T^4$, and that the gauge coupling constant at the scale below M_i has a logarithmic dependence on the scale M_i , one finds the free energy of ϕ , so-called “*thermal log*” potential as [9]:

$$V_{\text{th-log}} = a_L \alpha(T)^2 T^4 \ln [\lambda^2 |\phi|^2 / T^2] \quad (2.6)$$

where a_L is an order one constant.

Thus, the free energy that affects the dynamics of scalar condensation can be approximately parametrized as

$$V_{\text{th}} = \begin{cases} a_M \lambda^2 T^2 |\phi|^2 & \text{for } \lambda|\phi| < T \\ a_L \alpha(T)^2 T^4 \ln [\lambda^2 |\phi|^2 / T^2] & \text{for } \lambda|\phi| > T \end{cases} \quad (2.7)$$

where $a_{M/L}$ are order one constants. The effective potential for the scalar condensation is given by the sum: $V_{\text{eff}} = V + V_{\text{th}}$.

The scalar condensation begins to oscillate when the Hubble parameter becomes comparable to its effective mass:

$$H_{\text{os}} \simeq \max [m, \lambda T \text{ (for } \lambda\phi_i < T), \alpha T^2 / \phi_i \text{ (for } T < \lambda\phi_i)] \quad (2.8)$$

with the initial amplitude ϕ_i being assumed to be larger than v . Here we omit the order one constants $a_{M/L}$ for simplicity. There are three cases depending on which term dominates the effective potential: thermal log, thermal mass and zero temperature mass. The temperature T_{os} at the beginning of oscillation for each cases is summarized as follows [15]:

- The scalar ϕ begins to oscillate with thermal log potential if

$$\phi_i < \alpha T_R \sqrt{\frac{M_{\text{pl}}}{m}} \quad \text{and} \quad \phi_i > (T_R/\lambda)^{2/3} (\alpha M_{\text{pl}})^{1/3}. \quad (2.9)$$

The temperature at the beginning of oscillation is given by

$$T_{\text{os}} = \sqrt{\frac{\alpha M_{\text{pl}}}{\phi_i}} T_R. \quad (2.10)$$

In addition to above two inequalities, the condition $T_{\text{os}} > T_R$ should be met since otherwise the scalar oscillates with the zero-temperature mass term [case (iii)].

- The scalar ϕ begins to oscillate with thermal mass if

$$\begin{cases} \lambda > \left[\frac{m^3}{T_R^2 M_{\text{pl}}} \right]^{1/4} & \text{and} \quad \phi_i < (T_R/\lambda)^{2/3} (\alpha M_{\text{pl}})^{1/3} & \text{for } \lambda M_{\text{pl}} > T_R \\ \lambda > \sqrt{\frac{m}{M_{\text{pl}}}} & & \text{for } \lambda M_{\text{pl}} < T_R. \end{cases} \quad (2.11)$$

The temperature at the onset of oscillation is given by

$$T_{\text{os}} = \begin{cases} [\lambda T_R^2 M_{\text{pl}}]^{1/3} & \text{for } \lambda M_{\text{pl}} > T_R \\ \lambda M_{\text{pl}} & \text{for } \lambda M_{\text{pl}} < T_R. \end{cases} \quad (2.12)$$

- Otherwise, the scalar ϕ begins to oscillate with zero temperature mass, and the temperature is given by

$$T_{\text{os}} = \begin{cases} [m M_{\text{pl}} T_R^2]^{1/4} & \text{for } m M_{\text{pl}} > T_R^2 \\ \sqrt{m M_{\text{pl}}} & \text{for } m M_{\text{pl}} < T_R^2. \end{cases} \quad (2.13)$$

2.2.2 Non-perturbative particle production

The dispersion relations of (s)quarks depend on the scalar field value:

$$\omega_Q(t) = \sqrt{\lambda^2 |\phi(t)|^2 + m_{\text{eff},Q}^2 + \mathbf{k}^2} \quad (2.14)$$

with the soft (s)quark mass being neglected. Here $m_{\text{eff},Q}$ encodes the finite density correction to the real part of dispersion relation: *e.g.* the contribution from particles in the thermal plasma is imprinted as the thermal mass term. In our case, the initial amplitude ϕ_i is much larger than the scalar mass m : $\lambda \phi_i \gg m$, and hence perturbative decay is

kinematically forbidden during the most region of oscillation period. Then, the following non-perturbative particle production becomes important.

The non-perturbative particle production occurs when the adiabaticity of (s)quarks is broken down: $|\dot{\omega}_Q/\omega_Q^2| \gg 1$ [16, 17]. The condition for non-perturbative production is given by

$$\lambda\tilde{\phi} \gg \max \left[m, \frac{m_{\text{eff},Q}^2}{m} \right], \quad (2.15)$$

where $\tilde{\phi}$ is the amplitude of the oscillation of ϕ . Note that even if the back-reaction of particle production dominates the dynamics of scalar, the condition [Eq. (2.15)] is useful by simply replacing m with the effective mass \bar{m}_{eff} that encodes the finite density correction.

If Eq. (2.15) is met, after the first crossing of the region where the adiabaticity is broken: $|\phi| < \phi_{\text{NP}}$ with $\phi_{\text{NP}} \equiv (m\tilde{\phi}/\lambda)^{1/2}$, the number density of (s)quark suddenly acquires the value

$$n_Q \simeq \frac{k_*^3}{4\pi^3}; \quad k_* \equiv (\lambda m \tilde{\phi})^{1/2} \quad (2.16)$$

for one complex scalar or one chiral fermion [17]. Before the first crossing of the region where the adiabaticity is broken, we have $n_Q = 0$. Thus, from Eq.(2.15), we obtain the condition for the non-perturbative production:

$$k_*^2 \gg \max [m^2, g^2 T^2]. \quad (2.17)$$

In parameters of our interest,^{#1} the non-perturbative production may occur when the scalar ϕ begins to oscillate with the zero-temperature mass, and the reheating takes place after the beginning of oscillation. Hence, at the first crossing, the latter condition of Eq. (2.17) implies

$$T_R < (\lambda/g^2) \sqrt{\frac{m}{M_{\text{pl}}}} \phi_i. \quad (2.18)$$

Then there are three cases.

- (A) ϕ begins to oscillate with thermal log potential.
- (B) ϕ begins to oscillate with zero-temperature potential without non-perturbative particle production.
- (C) ϕ begins to oscillate with zero-temperature potential with efficient non-perturbative particle production.

We discuss the scalar trapping in these three cases separately.

In Fig. 1, we show the contours of constant T_{os} on ϕ_i vs. T_R plane; here, we used $m = 1$ GeV, $\lambda = 0.05$, and α of the strong interaction. The shaded region (A) corresponds

^{#1} Here we assumed $\lambda \lesssim g^2$. See also footnote #2.

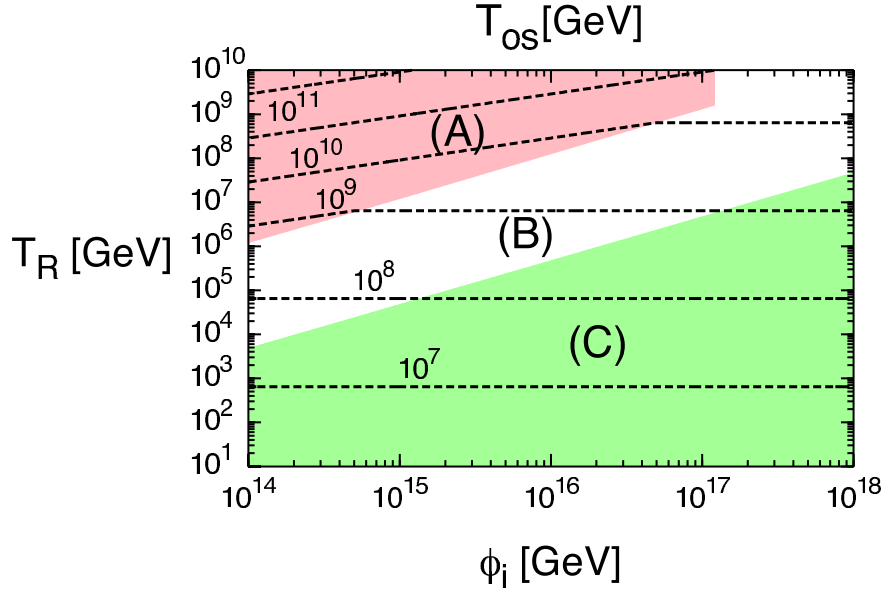


Figure 1: The contour plot of T_{os} as a function of ϕ_i and T_R . Here, we have taken $m = 1$ GeV, $\lambda = 0.05$, and used α of the strong interaction. In the region (A), the scalar begins to oscillate with thermal log potential. In the region (B), the scalar begins to oscillate with zero-temperature mass without non-perturbative particle production. In the region (C), the scalar begins to oscillate with zero-temperature mass with efficient non-perturbative particle production.

to the parameters where the scalar begins to oscillate with the thermal log term. On the other hand, in the regions (B) and (C), the scalar begins to oscillate with the zero temperature mass term. The boundary between (A) and (B) [(B) and (C)] is given by Eq.(2.9) [Eq. (2.18)]. Note that the region where the scalar starts to oscillate with the thermal mass term lies outside of this figure: higher T_R and smaller ϕ_i . We will not consider this case in the following.

The subsequent evolution of scalar/plasma system after the first crossing crucially depends on the setup. There are two cases:

- (i). If the produced particles cannot decay faster than the oscillation period, the parametric resonance can occur due to the induced emission effect of previously produced particles. Hence, the number density grows exponentially and then the back-reaction soon dominates the effective mass of ϕ . In addition, because of the largeness of the number density of Q in the background, the linear potential of ϕ is induced. [See Eq. (2.23).] This case will be extensively studied in Sec. 3.2.
- (ii). On the other hand, if the non-perturbatively produced (s)quarks can decay into other light particles much faster than the oscillation period of ϕ , then the phenomena like instant preheating [20] take place. In this case, the linear potential generated by

the produced particles is insignificant since the produced particles soon decay. See Sec. 3.3 for further discussion.

First, let us consider the case (i). After the passage of non-adiabatic region $|\phi| < \phi_{\text{NP}}$, the produced particles become heavy due to large field value $|\phi(t)|$ and their decay rate become large correspondingly. If the decay rate $\Gamma_{\text{dec}}^Q \sim \kappa^2 \lambda |\phi(t)|$ [with κ being the mixing parameter given in Eq. (2.3)] reaches $\Gamma_{\text{dec}}^Q(t_{\text{dec}})t_{\text{dec}} \sim 1$ with $t_{\text{dec}} \ll m^{-1}$, the produced particles completely decay into SM radiation well before the scalar moves back to the origin. The condition for the case (i), which requires the stability of Q for the time scale of our interest, is given by

$$\kappa \ll \sqrt{\frac{m}{\lambda \tilde{\phi}}}. \quad (2.19)$$

If the above condition is satisfied, the number density of (s)quarks grows explosively due to the parametric resonance as ϕ crosses the region where the adiabaticity is broken. In our set up, the condition Eq. (2.15) implies

$$k_*^2 \gg \max \left[\bar{m}_{\text{eff}}^2, g^2 T^2, g^2 \frac{n_Q}{k_*}, \lambda^2 \frac{n_Q}{k_*} \right] \quad (2.20)$$

where \bar{m}_{eff} represents the effective mass for ϕ that includes the back-reaction of particle creation and it is estimated as

$$\bar{m}_{\text{eff}}^2 \sim \max \left[m^2, \lambda \frac{n_Q}{\phi} \right]. \quad (2.21)$$

Eventually, the parametric resonance stage finishes when the inequality Eq. (2.15) is saturated. This condition can be rewritten as

$$\bar{m}_{\text{eff}} \ll \min \left[k_*, \frac{\lambda^2}{g^2} k_* \right] \equiv \frac{k_*}{c}. \quad (2.22)$$

In the following, we consider the case: $\lambda \lesssim g^2$ for simplicity,^{#2} and hence $c = g^2/\lambda^2$. In addition, in the case (i), the linear potential of ϕ , which is obtained as [17]

$$V_{\text{linear}} \sim \lambda^2 |\phi|^2 \langle |\tilde{Q}|^2 \rangle \sim \lambda^2 |\phi|^2 \int \frac{d^3 k}{(2\pi)^3 \omega_Q} f_Q(k) \sim \lambda |\phi| n_Q \quad \text{for } |\phi| > \phi_{\text{NP}}, \quad (2.23)$$

is generated because sizable amount of Q exists irrespective of the field value of ϕ .^{#3} (Here, f_Q denotes the distribution function of Q .) Such a linear potential plays very important role in studying the evolution of ϕ , as we will see in the next section.

^{#2} For large λ , ϕ may be more likely to be trapped at the origin since the region (C) where the non-perturbative production occurs becomes larger [See Eq. (2.18)].

^{#3} Here we used the fact that the k integration is dominated at $k \sim k_*$ and $\omega_Q \sim \lambda |\phi|$. For the case of $|\phi| \lesssim \phi_{\text{NP}}$, $\omega_Q \sim k_*$, and hence the potential becomes quadratic in $|\phi|^2$.

Next, let us consider the case (ii), which corresponds to the case where the condition (2.19) is not satisfied. In this case, the produced particles soon decay at $|\phi(t_{\text{dec}})| \sim [m\tilde{\phi}/(\kappa^2\lambda)]^{1/2} \ll \tilde{\phi}$. Taking an oscillation time average, one finds the effective dissipation rate in this case [15]:

$$\Gamma_{\phi}^{(\text{dis})} \simeq N_{\text{d.o.f.}} \times \frac{\lambda^2 m}{2\pi^4 \kappa} \quad (2.24)$$

where $N_{\text{d.o.f.}}$ stands for the degree of freedom normalized by one complex scalar or one chiral fermion. In addition, in case (ii), the parametric resonance does not occur and the linear potential generated by the produced particles is insignificant.

3 Saxion Dynamics

The strong CP problem is one of the serious theoretical problems in the SM. The PQ mechanism is a promising solution to the strong CP problem [2]. It utilizes a global U(1) symmetry, called PQ symmetry, which is spontaneously broken at the scale f_a . The axion appears as a consequence of spontaneous breakdown of the PQ symmetry and it plays an important role in cosmology and phenomenology [21, 22].

In SUSY extensions of the axion model, the early Universe cosmology becomes much more non-trivial because of the presence of flat direction in the scalar potential, called saxion [23, 24, 25, 26]. The cosmological evolution of the saxion field depends on the detailed structure of the saxion potential. Among various possibilities (see Ref. [22] for a partial list of the stabilization mechanism), in this paper we focus on the case where there is only one (complex) PQ scalar field having VEV [27, 28, 29, 30].

The potential of the saxion field ϕ is lifted only by the effect of SUSY breaking and is very flat for $|\phi| \gg f_a$. Then, without a tuning, the saxion is expected to be initially placed at the origin ($|\phi| = 0$) or around the Planck scale ($|\phi| \sim M_{\text{pl}}$), depending on whether the saxion receives positive or negative Hubble mass correction during inflation. In the former case, the saxion likely causes thermal inflation [6, 7, 8] as extensively studied in Refs. [31, 32, 33, 34, 35]. Once thermal inflation happens, the saxion must successfully reheat the Universe before the big-bang nucleosynthesis (BBN) begins. Unfortunately, the saxion often dominantly decays into the axion pair, which would invalidate the thermal inflation scenario. (However, it is possible that the saxion dominantly decays into the Higgs sector in the DFSZ model [36].)

On the other hand, the case of large initial amplitude of the saxion, $|\phi| \gg f_a$, has not been studied in detail so far. We mainly focus on this case and investigate its cosmological effects. The dynamics of the saxion in such a case is rather complicated due to two effects: thermal effects and non-perturbative particle production.^{#4} As will become clear, both effects tend to temporarily stabilize the saxion around the origin, $|\phi| = 0$. We will show

^{#4} Thermal effects on the saxion dynamics were considered in Refs. [37, 14] in a different class of PQ model.

Table 1: Charge assignments

	$\hat{\phi}$	\hat{Q}	$\hat{\bar{Q}}$
U(1) _{PQ}	+1	-1	0
SU(3) _c	1	3	$\bar{3}$

that, even if we start with a large saxion initial amplitude, it will be eventually trapped at the origin due to these effects and thermal inflation takes place for the most reasonable parameter choices. Cosmological issues related to this will be discussed in Sec. 4.

3.1 Setup

Amongst several ways to stabilize the saxion, we focus on the models with only one PQ scalar whose potential is stabilized by the quadratic term from SUSY breaking [27, 28, 29, 30].

To capture the essential features of saxion dynamics within these models, we approximate the saxion potential as

$$V \sim \begin{cases} -m_s^2 (|\phi|^2 - M^2) & \text{for } |\phi| < M \\ -m_s^2 M^2 \log^3(|\phi|/M) & \text{for } M < |\phi| < f_a \\ +m^2 (|\phi|^2 - f_a^2) & \text{for } f_a < |\phi|. \end{cases} \quad (3.1)$$

where f_a is the PQ scale, m_s and m are mass parameters terms from SUSY breaking effect. (Note that we have the relation $m f_a \simeq m_s M$.) In addition, M is the messenger scale in the framework of gauge-mediation. (For the cases of gravity- and anomaly-mediation, see the discussion below.) The relation between m_s and m depends on the mechanism of SUSY breaking. In the models based on gauge-mediation [27, 38, 39, 40], m is expected to be of order the gravitino mass. Furthermore, the negative soft mass of saxion potential at its origin comes from the three loop effect via the Yukawa coupling λ , and hence it is related with the squark mass as $m_{\text{soft}} \propto m_s/\lambda$ (with m_{soft} being the sfermion masses in SUSY SM.) As a result, there can be a hierarchy between M and f_a . In the gravity- or anomaly-mediation models, the renormalization group effect may stabilize the saxion potential [28, 29]. If so, $m \simeq m_s$ (and $M \simeq f_a$).

We consider the SUSY extension of the hadronic axion model [41]. We introduce the following superpotential:

$$W = \lambda \hat{\phi} \hat{\bar{Q}} \hat{Q}, \quad (3.2)$$

where \hat{Q} and $\hat{\bar{Q}}$ are identified as the PQ quark superfields^{#5} charged under both $SU(3)_c$ and $U(1)_{PQ}$ so that it has $U(1)_{PQ}$ - $SU(3)_c$ - $SU(3)_c$ anomaly and solves the strong CP problem. The charge assignments are summarized in Table 1.^{#6} (Here, the PQ charges of \hat{Q} and $\hat{\bar{Q}}$ are arranged so that the mixings with the SM fermions are possible.)

With the PQ charges given in Table 1, there may exist the following terms in the superpotential

$$W_{\text{mix}} = \kappa \hat{\bar{Q}} \hat{q}_L \hat{H}_d, \quad (3.3)$$

which induce the mixing with the SM particles. (Here, all the particles in the SUSY SM are vanishing PQ charge.) If these mixing terms exist, PQ quarks decay into SM fields. As will become clear, the dynamics of saxion significantly depends on whether PQ (s)quarks are stable or not. In this paper we consider two cases separately: small mixing limit (Sec. 3.2) and large mixing limit (Sec. 3.3).

3.2 Saxion dynamics: small mixing case

As mentioned at the beginning of Sec. 2.2, the saxion may be trapped at the origin due to the thermal/non-thermal effects even though its initial field value is displaced far from the origin. Let us discuss thermal and non-thermal effects in turn and derive the condition for saxion trapping. In this subsection we focus on the case of small mixing limit between PQ and SM quarks, and hence PQ (s)quarks are considered as stable particles.

3.2.1 Case (A): Thermal trapping

First, we consider the case (A). If the negative mass of saxion potential at the origin vanishes due to the thermal effects when the Hubble parameter becomes comparable to the dissipation rate of saxion into radiation, the saxion is trapped at the origin. In our setup, the dissipation rate is expected to become larger than the Hubble parameter soon after the onset of oscillation because the temperature T_{os} is much larger than m and the coupling λ is not so small. Hence, we derive the condition for saxion trapping with the approximation that the saxion immediately dissipates its energy after its onset of oscillation. Note that there is no explosive particle production due to the non-perturbative effect for $\lambda < \alpha$.

Let us consider the first crossing of $\phi = 0$. The time scale δt , during which the saxion passes through $|\phi| \lesssim \phi_c \equiv T_{\text{os}}/\lambda$, is estimated as $\delta t \sim 1/(\lambda \alpha T_{\text{os}})$. The abundance of PQ (s)quarks that is produced during this time interval is given by $n_Q \sim n_{\text{th}}^2 \langle \sigma_{\text{prod}} v \rangle \delta t$, where $\langle \sigma_{\text{prod}} v \rangle \sim g^4/T_{\text{os}}^2$ denotes the PQ (s)quark pair production cross section and $n_{\text{th}} \sim T_{\text{os}}^3$ is

^{#5} Symbols with hats represent the superfield. The corresponding scalar component is represented by symbols without hats.

^{#6} There can also be PQ leptons, \hat{L} and $\hat{\bar{L}}$, which are combined with PQ quarks to form $SU(5)$ fundamental and anti-fundamental representations as (\hat{Q}, \hat{L}) and $(\hat{\bar{Q}}, \hat{\bar{L}})$. Even with those particles, the following arguments are unchanged.

the number density of particles in thermal bath. Thus the number density after the first passage for $|\phi| < \phi_c$ is evaluated as

$$n_Q^s \simeq T_{\text{os}}^3, \quad (3.4)$$

as long as $\lambda < \alpha$. (Here, the superscript “s” is for the saturated value.) After the passage, the saxion may climb up the linear potential $V_{\text{eff}} \simeq \lambda T_{\text{os}}^3 |\phi|$ for $|\phi| > \phi_c$, and eventually reach its maximum value in $\delta t_{\text{max}} \sim \alpha/(\lambda T)$. After the decoupling of PQ squarks at $|\phi| > \phi_c$, the annihilation of PQ squarks may still be efficient. In this case, their abundance reduces to $n_Q \sim (\langle \sigma_{\text{ann}} v \rangle \delta t_{\text{max}})^{-1}$. The result for $|\phi| > \phi_c$ is

$$n_Q^l \simeq \min \left[1, \frac{\lambda}{\alpha^3} \right] \times T_{\text{os}}^3 \equiv d_\lambda T_{\text{os}}^3. \quad (3.5)$$

Recalling that we are considering the case where PQ (s)quarks are (quasi-)stable, we obtain the effective potential of the saxion:^{#7}

$$V_{\text{eff}} \simeq \begin{cases} \frac{\lambda n_Q^s}{\phi_c} |\phi|^2 & \text{for } |\phi| < \phi_c \\ \lambda n_Q^l |\phi| & \text{for } |\phi| > \phi_c. \end{cases} \quad (3.6)$$

Then, the conservative condition for saxion trapping is summarized as follows:^{#8}

- For $T_{\text{os}} > \lambda M$, the saxion can be stabilized at the origin solely by the effective mass term. The condition is nothing but $m_s^2 < \lambda n_Q^s / \phi_c$. This condition is rewritten as

$$T_{\text{R}} > m_s \left(\frac{\phi_i}{\lambda^2 \alpha M_{\text{pl}}} \right)^{1/2} \sim 10^3 \text{ GeV} \left(\frac{1}{\alpha} \right)^{1/2} \left(\frac{m_s / \lambda}{1 \text{ TeV}} \right) \left(\frac{\phi_i}{10^{18} \text{ GeV}} \right)^{1/2}. \quad (3.7)$$

- For $T_{\text{os}} < \lambda M$, the combination of the effective mass and linear term stabilizes the saxion at the origin. The condition is given by $m_s^2 M^2 < \lambda n_Q^l M$. This is rewritten as

$$\begin{aligned} T_{\text{R}} &> \left(\frac{m_s^2 M}{\lambda d_\lambda} \right)^{1/3} \left(\frac{\phi_i}{\alpha M_{\text{pl}}} \right)^{1/2} \\ &\sim 10^4 \text{ GeV} \left(\frac{1}{d_\lambda^{1/3} \alpha^{1/2}} \right) \left(\frac{m_s / \lambda}{1 \text{ TeV}} \right)^{1/3} \left(\frac{m f_a}{10^9 \text{ GeV}^2} \right)^{1/3} \left(\frac{\phi_i}{10^{18} \text{ GeV}} \right)^{1/2}. \end{aligned} \quad (3.8)$$

Note that these conditions are met for most parameters of our interest if the saxion oscillates with the thermal effects (recall that Eq.(2.9): $\alpha T_{\text{R}} > \phi_i \sqrt{m/M_{\text{pl}}}$ in the case (A)). The saxion oscillation around the effective potential soon decays because the effective dissipation rate, $\Gamma_\phi^{(\text{dis})}$, is large: $\Gamma_\phi^{(\text{dis})} \sim \lambda^2 \alpha T$. Eventually, the saxion is trapped at the origin.

^{#7} V_{eff} can be evaluated as V_{linear} given in Eq. (2.23). Note that, in the present case, the k integration is dominated at $k \sim T$.

^{#8}Throughout this paper, we impose the following conservative condition; we require that, for the trapping at the origin, the origin be the absolute minimum of the thermal potential. We do not consider the case where there are other minima than the origin. As we will see, even with this conservative criteria, the saxion is likely trapped for most parameters of our interest.

3.2.2 Case (B): Quasi-thermal trapping

Next, we consider the case (B). In this case, the thermal effects on the saxion potential is not important at the beginning of oscillation. Still, however, the situation is similar to the case (A) studied above. When the saxion passes through the origin, the thermal bath creates PQ (s)quarks. To see this, let us estimate the time scale δt , during which the saxion passes through the region $|\phi| \lesssim \phi_c \equiv T_{\text{os}}/\lambda$. It is estimated as $\delta t \sim T_{\text{os}}/k_*^2$ where $k_* = \sqrt{\lambda m \phi_i}$. The number density of PQ (s)quarks produced during this time interval is given by $n_Q \sim n_{\text{th}}^2 \langle \sigma_{\text{prod}} v \rangle \delta t$, and it is estimated as

$$n_Q^s \simeq \min \left[1, \frac{g^2}{\epsilon} \right] \times T_{\text{os}}^3 \simeq d_\epsilon T_{\text{os}}^3, \quad (3.9)$$

where $\epsilon \equiv k_*^2/(g^2 T_{\text{os}}^2)$ and $d_\epsilon \equiv (1 + \epsilon/g^2)^{-1}$. Note that $\lambda < \epsilon < 1$ in the case (B). If $\epsilon > g^2$, PQ (s)quarks does not reach thermal equilibrium during δt and hence the annihilation of PQ (s)quarks is neglected after the passage of $|\phi| \simeq \phi_c$. If $\epsilon < g^2$, PQ (s)quarks are thermally populated and their pair annihilation takes place after the saxion climbs up the potential around $|\phi| \sim \phi_c$. As a result, the number density of PQ (s)quark is given by

$$n_Q^l \simeq d'_\epsilon T_{\text{os}}^3, \quad (3.10)$$

where d'_ϵ encodes the model dependent numerical factor: $d'_\epsilon \sim g^2/\epsilon$ for $\epsilon > g^2$, and $d'_\epsilon \sim \lambda^2/(\epsilon \alpha^3)$ for $\epsilon < g^2$ and $\epsilon^2 \alpha^2 < \lambda^2 < \epsilon \alpha^3$. The resulting effective potential of the saxion is same as (3.6).

Then, the condition for saxion trapping is summarized as follows:

- For $T_{\text{os}} > \lambda M$, the saxion can be stabilized at the origin solely by the effective mass term. The condition is nothing but $m_s^2 < \lambda n_Q^s/\phi_c$. This condition is rewritten as

$$T_{\text{R}} > \frac{m_s^2}{\lambda^2 d_\epsilon \sqrt{m M_{\text{pl}}}} \sim 10^{-3} \text{ GeV} \left(\frac{1}{d_\epsilon} \right) \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^2 \left(\frac{m}{1 \text{ GeV}} \right)^{-1/2}. \quad (3.11)$$

- For $T_{\text{os}} < \lambda M$, the combination of the effective mass and linear term stabilizes the saxion at the origin. The condition is given by $m_s^2 M^2 < \lambda n_Q^l M$. This is rewritten as

$$\begin{aligned} T_{\text{R}} &> \left(\frac{m_s}{\lambda d'_\epsilon} \right)^{2/3} \frac{(m^{1/4} f_a)^{2/3}}{\sqrt{M_{\text{pl}}}} \\ &\sim 10^{-1} \text{ GeV} d'^{-2/3}_\epsilon \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^{2/3} \left(\frac{m}{1 \text{ GeV}} \right)^{1/6} \left(\frac{f_a}{10^9 \text{ GeV}} \right)^{2/3} \end{aligned} \quad (3.12)$$

for $m M_{\text{pl}} > T_{\text{R}}^2$. Note that these conditions are met for most parameters of our interest. Similar to the case (A), the saxion oscillation around the effective potential soon loses its energy due to the dissipation and is trapped at the origin.

3.2.3 Case (C): Non-thermal trapping

Finally we consider the case (C) ($k_* > gT_{\text{os}}$). Even if the above thermal effects are not efficient, the saxion can be trapped at the origin due to the explosive production of PQ squarks when the saxion passes through its origin. Let us see this phenomenon, which was dubbed as “non-thermal phase transition” [4].

As discussed in Sec. 2.2.2, PQ squarks are efficiently produced for $k_* > gT_{\text{os}}$ for each passage of $|\phi| = 0$ and its number density exponentially grows until the condition $k_*^2 \simeq m_{\text{eff},Q}^2$ is saturated. After the saturation, the amplitude of ϕ is given by

$$\tilde{\phi}_s \simeq c \sqrt{\frac{m\phi_i}{\lambda}}. \quad (3.13)$$

[For the definition of c , see Eq. (2.22).] Using this, the number density of PQ squark is given by $n_Q \sim \lambda \tilde{\phi}_s^3 / c^4$. The saxion effective potential is given by

$$V_{\text{eff}} \simeq \begin{cases} m_{\text{eff}}^2 |\phi|^2 & \text{for } |\phi| < \phi_{\text{NP}} \\ \lambda n_Q |\phi| & \text{for } |\phi| > \phi_{\text{NP}}, \end{cases} \quad (3.14)$$

where $\phi_{\text{NP}} = k_*/\lambda \sim \tilde{\phi}_s/c$ and $m_{\text{eff}}^2 = \lambda^2 n_Q / k_* \sim \lambda^2 \tilde{\phi}_s^2 / c^3$. Here $k_*^2 = \lambda \bar{m}_{\text{eff}} \tilde{\phi}_s$ with $\bar{m}_{\text{eff}}^2 = \lambda n_Q / \tilde{\phi} \sim \lambda^2 \tilde{\phi}_s^2 / c^4$. In this case, the squark pair annihilation rate is given by $\Gamma_{\text{ann},Q} = \langle \sigma_{\text{ann}} v \rangle n_Q$. Since it is dominated by $\phi(t) \sim \phi_{\text{NP}}$, the averaged annihilation rate is estimated as

$$\bar{\Gamma}_{\text{ann},Q} = \Gamma_{\text{ann},Q}(\phi_{\text{NP}}) \frac{\phi_{\text{NP}}}{\tilde{\phi}_s} = \frac{g^4}{\lambda c^4} \frac{\tilde{\phi}_s^2}{\phi_{\text{NP}}} \sim \frac{g^4}{\lambda c^3} \tilde{\phi}_s \sim \bar{m}_{\text{eff}} \frac{g^4}{\lambda^2 + g^2}. \quad (3.15)$$

Thus the pair annihilation of squarks is not efficient in the time scale of saxion oscillation.^{#9}

Then, the condition for saxion trapping is summarized as follows:

- For $\phi_{\text{NP}} > M$, the saxion can be stabilized at the origin solely by the mass term. The condition is nothing but $m_s < m_{\text{eff}}$. This condition is rewritten as

$$\phi_i > \frac{cm_s^2}{\lambda m} \sim 10^6 \text{ GeV } c\lambda \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^2 \left(\frac{m}{1 \text{ GeV}} \right)^{-1}. \quad (3.16)$$

^{#9} The annihilation of PQ squarks may become efficient slightly before the condition $k_* \sim m_{\text{eff},Q}$ is saturated, since g is not small and the rate of parametric resonance $\mu \bar{m}_{\text{eff}}$ is smaller than \bar{m}_{eff} . Even if this is the case, the conclusion is almost unchanged: Actually, in this case, the number density of squark stops growing slightly before the saturation and the saxion loses its energy in each oscillation via the annihilation. The annihilation of PQ squarks produces background plasma. Thus, it is considered that the saxion oscillation soon disappears within a few Hubble time and that it is eventually trapped at the origin. [See the following discussion in the text below Eq. (3.21).]

- For $\phi_{\text{NP}} < M$, the combination of mass and linear terms stabilizes the saxion at the origin. The condition is given by $m_s^2 < \lambda n_Q/M$. This is rewritten as

$$\phi_i > \frac{(cm_s^2 M)^{2/3}}{\lambda^{1/3} m} \sim 10^8 \text{ GeV } c^{2/3} \lambda^{1/3} \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^{2/3} \left(\frac{m}{1 \text{ GeV}} \right)^{-1/3} \left(\frac{f_a}{10^9 \text{ GeV}} \right)^{2/3}. \quad (3.17)$$

These conditions are likely satisfied for parameters of our interest.

Here we study the subsequent evolution of the system after the saxion trapping. After the saturation, the saxion loses its energy through the squark pair annihilation into radiation.^{#10} The energy loss in one oscillation of the saxion is estimated as

$$\Delta\rho_\phi^{(Q-\text{ann})} = m_Q \langle \sigma_{\text{ann}} v \rangle n_Q^2 \Delta t \sim \frac{g^4}{\lambda^2} \bar{m}_{\text{eff}} n_Q \sim \frac{g^4}{c^6} \tilde{\phi}^4, \quad (3.18)$$

where $\Delta t = \bar{m}_{\text{eff}}^{-1}$ using the fact that the energy loss is dominated around $\phi \sim \tilde{\phi}$. We can define the effective dissipation rate of the saxion through

$$\bar{\Gamma}_\phi^{(Q-\text{ann})} = \frac{\Delta\rho_\phi^{(Q-\text{ann})}}{\rho_\phi} \bar{m}_{\text{eff}} \sim \frac{g^4}{\lambda c^4} \tilde{\phi}. \quad (3.19)$$

Since this is much larger than the Hubble expansion rate ($H_{\text{os}} \sim m$) with $\tilde{\phi} \sim \tilde{\phi}_s$, the produced PQ squarks efficiently annihilate and correspondingly the saxion amplitude decreases. By solving the equation

$$\dot{\rho}_\phi + \bar{\Gamma}_\phi^{(Q-\text{ann})} \rho_\phi = 0, \quad (3.20)$$

we find

$$\tilde{\phi}(t) = \left[\tilde{\phi}_s^{-1} + \frac{g^4}{4\lambda c^4} (t - t_i) \right]^{-1}. \quad (3.21)$$

Thus it is seen that the amplitude $\tilde{\phi}$ decays in a time scale $(\delta t)_{\text{ann}} \sim \lambda c^4 / (g^4 \tilde{\phi}_s)$, which is much shorter than the Hubble time scale. The amplitude follows Eq. (3.21) as long as the condition for the non-perturbative particle production is saturated ($k_* \sim m_{\text{eff},Q}$). The annihilation of PQ squarks produces background plasma, which would give a large effective mass to the PQ squarks and terminate the parametric resonance. Actually, after a time scale $(\delta t)_{\text{ann}}$, the radiation with a “would be” temperature $T^{(\text{w.b.})} \sim \rho_\phi^{1/4} \sim \sqrt{m\phi_i}$ is produced. (Note that one has $T^{(\text{w.b.})} \sim k_*/\lambda^{1/2} > (g/\lambda^{1/2})T > T$.) After that, the produced plasma soon thermalizes, and the condition for the non-perturbative particle

^{#10} Since the produced squarks are highly correlated and have non-perturbatively large occupation number, the quasi-particle treatment may not be justified soon after the end of preheating, and other processes including in-medium-bremsstrahlung and multi-annihilation may also contribute. Here, however, we simply estimate the energy transportation of saxion to the radiation by the pair annihilation of quasi-particle squarks.

production is saturated by the effective mass term for the PQ squark, as $k_*^2 \sim g^2 T^2$. Similar to the above estimates, the saxion also efficiently dissipates its energy into radiation in this case. Therefore it is considered that the saxion oscillation soon disappears within a few Hubble time around the steep effective potential. Since the temperature is much larger than the soft mass scale m_s , the saxion is eventually trapped by thermal mass term even if the reheating temperature is extremely low.

After all, for all the cases (A)–(C), the saxion is likely trapped for most parameters of our interest.

3.3 Saxion dynamics: large mixing case

Now let us study the case of large mixing between PQ and SM quarks, so that PQ (s)quarks are unstable. Below we analyze the dynamics of the saxion in the cases (A) – (C) separately as in the small mixing case.

3.3.1 Case (A)

In the case (A), the saxion begins to oscillate with thermal log potential. As explained in Sec. 3.2.1, the time scale required for the saxion to pass through $|\phi| \lesssim \phi_c = T_{\text{os}}/\lambda$ is estimated as $\delta t \sim 1/(\lambda\alpha T_{\text{os}})$. The number density PQ (s)quarks produced from thermal plasma in this time scale is given by $n_Q \sim n_{\text{th}}^2 \langle \sigma_{\text{prod}} v \rangle \delta t \sim \kappa^2 T_{\text{os}}^3/\lambda$. (In the following, we assume $\kappa \sim 1$.) Thus PQ (s)quarks are expected to be thermally populated. After the passage, the saxion may climb up the linear potential $V_{\text{eff}} = \lambda T_{\text{os}}^3 |\phi|$ for $|\phi| > \phi_c$, and eventually reach its maximum value in $\delta t_{\text{max}} \sim \alpha/(\lambda T)$. If the decay time is longer than this time scale, the saxion moves back to its origin by the linear potential and consequently oscillates with the linear potential. Otherwise, the produced (s)quarks soon decay and the saxion oscillates with the thermal log potential. This condition is given by $\kappa^2 > \lambda/\alpha$ and we concentrate on this case unless otherwise stated.

Note that the effective dissipation rate of the saxion in the regime of thermal log potential is given by $\Gamma_{\phi}^{(\text{dis})} \sim b\alpha^2 T^3/(\tilde{\phi}\phi_c)$ and hence we obtain $\Gamma_{\phi}^{(\text{dis})}/\bar{m}_{\text{eff}} \sim \lambda\alpha$. It means that the saxion oscillation soon dissipates its energy once it starts to oscillate around the origin unless λ is very small. This is because the effective dissipation rate decrease more slowly than the Hubble expansion rate. As a result, the saxion is trapped at the origin.

Thus, the condition for the saxion trapping is summarized as follows.

- For $T_{\text{os}} > \lambda M$, the saxion can be stabilized at the origin solely by the thermal mass term for $m_s < \lambda T_{\text{os}}$. This condition is rewritten as

$$T_{\text{R}} > \frac{m_s}{\lambda} \left(\frac{\phi_i}{\alpha M_{\text{pl}}} \right)^{1/2} \sim 10^3 \text{ GeV} \left(\frac{1}{\alpha^{1/2}} \right) \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right) \left(\frac{\phi_i}{10^{18} \text{ GeV}} \right)^{1/2}. \quad (3.22)$$

- For $T_{\text{os}} < \lambda M$, the combination of the thermal mass and the linear potential/the thermal log stabilizes the saxion at the origin. The condition is given by $m_s <$

$\alpha T_{\text{os}}^2/M$. This is rewritten as

$$T_{\text{R}} > \left(\frac{m f_a \phi_i}{\alpha^2 M_{\text{pl}}} \right)^{1/2} \sim 10^6 \text{ GeV} \left(\frac{0.1}{\alpha} \right) \left(\frac{m f_a}{10^{10} \text{ GeV}^2} \right)^{1/2} \left(\frac{\phi_i}{10^{18} \text{ GeV}} \right)^{1/2}. \quad (3.23)$$

Here we have omitted the case $\lambda/\alpha < \kappa^2$ since it is the same as Eq. (3.8) for $d_\lambda = 1$.

3.3.2 Case (B)

In the case (B), the saxion begins to oscillate with zero-temperature mass. As explained in Sec. 3.2.2, the time interval required for the saxion to pass through the region $|\phi| \lesssim \phi_c = T_{\text{os}}/\lambda$ is estimated as $\delta t \sim T_{\text{os}}/k_*^2$. The number density PQ (s)quarks produced from thermal plasma in this time period is given by $n_Q \sim n_{\text{th}}^2 \langle \sigma_{\text{prod}} v \rangle \delta t \sim (\kappa^2 \alpha T_{\text{os}}^2/k_*^2) T_{\text{os}}^3$. By noting $(1/\lambda^2 \gtrsim) \alpha T_{\text{os}}^2/k_*^2 \gtrsim 1$ in the case (B), we find that PQ (s)quarks are expected to be thermally populated in this time interval. For most parameters of our interest, the linear potential may soon decay, and hence the effective potential for the saxion near the origin is given by thermal potential.^{#11}

The effective dissipation rate of the saxion in the regime is estimated as $\Gamma_\phi^{(\text{dis})} \sim \lambda \alpha T^2/\tilde{\phi}$ and hence we obtain $\Gamma_\phi^{(\text{dis})}/m \sim \lambda \alpha T_{\text{os}}^2/(m \tilde{\phi})$. It leads to $\lambda^2 < \Gamma_\phi^{(\text{dis})}/m < 1$. Thus the saxion oscillation soon dissipates its energy unless λ is very small since $\Gamma_\phi^{(\text{dis})}$ decreases more slowly than the Hubble rate.

In summary, the condition for the saxion trapping is as follows:

- For $T_{\text{os}} > \lambda M$, the saxion can be stabilized at the origin solely by the effective mass term for $m_s < \lambda T_{\text{os}}$. This condition is rewritten as

$$T_{\text{R}} > \frac{1}{\lambda^2} \frac{m_s^2}{\sqrt{m M_{\text{pl}}}} \sim 10^{-3} \text{ GeV} \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^2 \left(\frac{m}{1 \text{ GeV}} \right)^{-1/2}. \quad (3.24)$$

- For $T_{\text{os}} < \lambda M$, the combination of the effective mass and thermal log stabilizes the saxion at the origin. The condition is given by $m_s < \alpha T_{\text{os}}^2/M$. This is rewritten as

$$T_{\text{R}} > \frac{f_a}{\alpha} \left(\frac{m}{M_{\text{pl}}} \right)^{1/2} \sim 10 \text{ GeV} \left(\frac{0.1}{\alpha} \right) \left(\frac{f_a}{10^9 \text{ GeV}} \right) \left(\frac{m}{1 \text{ GeV}} \right)^{1/2}. \quad (3.25)$$

3.3.3 Case (C)

In the case (C), the non-perturbative production occurs at the crossing of $\phi \sim 0$. As explained in Sec. 2.2.2, the number density of PQ (s)quarks produced in this way is given

^{#11} In fact, the linear potential can decay if $1 < \Gamma_{\text{dec}}^Q \delta t$ with $\Gamma_{\text{dec}}^Q \sim \kappa^2 \lambda |\phi|$ and $\delta t \sim m \tilde{\phi}/(\lambda T^3)$. Such a condition is likely satisfied because $\Gamma_{\text{dec}}^Q \delta t \sim (\kappa^2/\lambda)(m \tilde{\phi}/T^2) > \kappa^2 \alpha/\lambda$.

by $n_Q \sim k_*^3/(4\pi^3)$. The produced PQ (s)quarks decay at $|\phi(t_{\text{dec}})| \sim [m\tilde{\phi}/\lambda\kappa^2]^{1/2}$. From this, we can estimate the effective dissipation rate of the saxion as [15]

$$\Gamma_\phi^{(\text{dis})} \simeq N_{\text{d.o.f.}} \times \frac{\lambda^2 m}{2\pi^4 \kappa}. \quad (3.26)$$

Due to this effect, thermal plasma with temperature $T \sim \sqrt{\lambda m \phi_i / g^{1/2}}$ is produced within one oscillations of the saxion. This process continues until the condition for the non-perturbative production is violated. Actually the condition is soon violated after a few Hubble time after the oscillation, that is, $gT_{\text{NP}} \sim k_*^{\#12}$ [See Eq. (2.17)]. Then, thermal effects tend to stabilize the saxion at the origin, similar to the case (A) and (B).

By simply assuming that the temperature of the plasma at the end of non-perturbative particle production is given by $T_{\text{NP}} \sim \sqrt{\lambda m \phi_i} / g$, the condition for the saxion trapping is summarized as follows.

- For $T_{\text{NP}} > \lambda M$, the saxion can be stabilized at the origin solely by the effective mass term for $m_s/\lambda < T_{\text{NP}} \sim \sqrt{\lambda m \phi_i} / g$.
- For $T_{\text{NP}} < \lambda M$, the combination of the effective mass and thermal log stabilizes the saxion at the origin. The condition is given by $m_s < \alpha T_{\text{NP}}^2 / M$. This condition is rewritten as $4\pi f_a < \lambda \phi_i$.

These conditions are likely satisfied in the most parameters of our interest. Then, as in the case of (A) and (B), the saxion soon dissipates its energy unless λ is quite small, and sits around its origin. Once the saxion is trapped at the origin, it potentially causes thermal inflation, as studied in the next section.

3.3.4 Comment on the case of non-trapped saxion

As studied above, it is possible that the saxion oscillates with an amplitude much larger than the PQ scale without trapping at the origin in the limit of small λ and T_{R} . Then it may be non-trivial in which minima the saxion relaxes and whether axionic domain walls forms or not. We briefly see what happens in this case.

Let us suppose that the saxion oscillates in the real axis in ϕ with an amplitude $\tilde{\phi}$ and define the axion as $a \equiv \sqrt{2}\text{Im}\phi$. First note that the mass of a changes from $+m^2$ to $-m^2$ around $|\phi| \simeq f_a$ and further it develops to $-m_s^2$ at $|\phi| \lesssim M$. In one oscillation of the saxion, the fluctuation along the axion direction develops due to the tachyonic instability as $\delta a/a \sim f_a/\tilde{\phi}$ at $|\phi| \lesssim f_a$. Therefore, the initial axion fluctuation sourced by inflationary quantum fluctuations significantly develops at $\tilde{\phi} \sim f_a$ and the motion of ϕ is expected to become chaotic in the complex plane. This may lead to the formation of axionic domain walls, even in the case of no explicit symmetry restoration.

^{#12} The interaction given in Eq. (3.3) also affects the thermal mass of Q . Even with such a contribution, the present discussion is unchanged because we are considering the case of $g \sim \kappa \sim 1$.

A further comment is in order. It was pointed out that the parametric resonant decay of the saxion into the axion leads to the nonthermal phase transition [42, 43, 44]. Note that these results were based on the potential $V \propto (|\phi|^2 - f_a^2)^2$. In our setup, the interaction term between the saxion and axion exists only in the logarithmic term and the parametric resonant enhancement of the axion would be much less efficient. Actually, the condition for the broad resonance is only marginally satisfied in our case. In order to make a definite conclusion whether the parametric resonance of the axion is efficient or not, we may need lattice calculations, which is beyond the scope of our analysis. However, regardless of the efficiency of the parametric resonance, we expect the formation of axionic domain walls as explained above.

4 Saxion Dynamics after Phase Transition

4.1 Condition for thermal inflation

In the previous section, we saw that the saxion is trapped at the origin $\phi = 0$ for almost all the cases even if the initial position of the saxion is displaced far from the origin. Now let us examine whether it causes thermal inflation or not.

After the trapping, the saxion begins to roll down the potential toward the minimum $|\phi| = f_a$ at the temperature $T = T_{\text{PT}} \simeq m_s/\lambda$. The condition for thermal inflation by the saxion to occur is $\rho_{\text{rad,inf}}(T = T_{\text{PT}}) < m_s^2 M^2 (\simeq m^2 f_a^2)$ where $\rho_{\text{rad,inf}}$ denotes the energy density of radiation or the inflaton oscillation, whichever is dominant.

First, let us consider the low-reheating temperature case: $T_{\text{PT}} > T_{\text{R}}$, so that the condition is given by $3H_{\text{PT}}^2 M_{\text{pl}}^2 < m_s^2 M^2$. Then, the radiation component in this case may contain two contributions: one from the dilute plasma due to the inflaton decay and the other from the dissipation of the saxion. The former (latter) is denoted by $\rho_r^{(\text{inf})}$ ($\rho_r^{(\phi)}$). If $\rho_r^{(\text{inf})}$ dominates the radiation energy density at $T = T_{\text{PT}}$, it is found that thermal inflation takes place if

$$T_{\text{R}} > \frac{T_{\text{PT}}^2}{\sqrt{m f_a}} \sim 10 \text{ GeV} \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^2 \left(\frac{1 \text{ GeV}}{m} \right)^{1/2} \left(\frac{10^{10} \text{ GeV}}{f_a} \right)^{1/2}. \quad (4.1)$$

On the other hand, if $\rho_r^{(\phi)}$ dominates the radiation energy density at $T = T_{\text{PT}}$, by noting that $\rho_r^{(\phi)}(T_{\text{PT}}) \simeq m^2 \phi_i^2 (H_{\text{PT}}/m)^{8/3}$, the condition is given by

$$f_a > \frac{M_{\text{pl}}(m_s/\lambda)^{3/2}}{(m \phi_i)^{3/4}} \sim 10^9 \text{ GeV} \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^{3/2} \left(\frac{1 \text{ GeV}}{m} \right)^{3/4} \left(\frac{M_{\text{pl}}}{\phi_i} \right)^{3/4}. \quad (4.2)$$

Second, let us consider the radiation dominant case: $T_{\text{PT}} < T_{\text{R}}$. Then the condition is given by $T_{\text{PT}}^4 < m_s^2 M^2$. This condition is rewritten as

$$f_a > \frac{1}{m} \left(\frac{m_s}{\lambda} \right)^2 \sim 10^6 \text{ GeV} \left(\frac{1 \text{ GeV}}{m} \right) \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^2. \quad (4.3)$$

If either of these conditions are met, thermal inflation is caused by the saxion. Actually, unless the reheating temperature is very low or the mass m is small, thermal inflation likely takes place. Otherwise, the saxion exits the thermal trapping potential before it begins to dominate the Universe and the cosmological problems associated with the axion overproduction from the saxion decay, as studied in detail in the next subsection, is alleviated. Even in such a case, however, the saxion may eventually dominate the Universe before it decays. In the following, we mainly consider the case of thermal inflation where the problem is most prominent.

4.2 Saxion dynamics after phase transition

Now let us see the cosmological evolution after the saxion begins to roll down from the high-temperature minimum $\phi = 0$ toward the true minimum $|\phi| = f_a$. For clarity, we assume $T_R > T_{PT}$ hereafter; this is the case in the most of parameters of our interest. We divide the saxion dynamics into two stages. At the first stage, the lower edge of the saxion oscillation field value is smaller than M . At the second stage, the lower edge of the saxion oscillation field value is larger than M .

4.2.1 First stage

We consider the dynamics of the saxion after it exits the thermal trapping potential at $T = T_{PT} \sim m_s/\lambda$. Let us define the time t so that $t = 0$ [$T(t = 0) = T_{PT}$] as the initial condition and reaches $|\phi| = M(f_a)$ at $t = t_M(t_f)$. Then, we can evaluate the dissipative effect on the saxion during its one oscillation around the minimum, using the fact that the background temperature is (almost) unchanged for the time scale of one oscillation.

- $t < t_M$

The evolution of the saxion is given by $|\phi(t)| = \phi_0 \exp(m_s t)$ where ϕ_0 is expected to be of order T (although its precise value is not important). Thus t_M is given by $t_M = m_s^{-1} \log(M/\phi_0)$. The energy loss of saxion due to the dissipation effect when it passes through the field value around ϕ is given by^{#13}

$$\left[\Delta \rho_\phi^{(\text{dis})} \right]_{0 < t < t_M} \sim \int_0^{t_M} dt \frac{1}{2} \dot{\phi}^2 \times \frac{b\alpha^2 T^3}{\phi^2} \sim b\alpha^2 T^3 m_s \log\left(\frac{M}{\phi_0}\right), \quad (4.4)$$

where $b\alpha^2 = 9\alpha^2/(128\pi^2 \ln \alpha^{-1}) \sim 3 \times 10^{-5}$ [45].

- $t_M < t < t_f$

After the passage of $|\phi| = M$, the saxion “speed” $\dot{\phi} (\sim m_s M)$ remains nearly constant

^{#13}The temperature $T \sim m_s/\lambda$ is slightly larger than the mass scale m_s , hence the use of dissipation rate in thermal background is marginally justified.

until it reaches $|\phi| = f_a$. Therefore, we have $|\phi(t)| = M[1 + m_s(t - t_M)]$ at this regime. We can evaluate the energy loss in a similar way as

$$\left[\Delta \rho_\phi^{(\text{dis})} \right]_{t_M < t < t_f} \sim \int_{t_M}^{t_f} dt \frac{1}{2} \dot{\phi}^2 \times \frac{b\alpha^2 T^3}{\phi^2} \sim b\alpha^2 T^3 m_s \quad (4.5)$$

- $t_f < t$

Finally, the saxion climbs up the potential at $|\phi| > f_a$ at $t = t_f \sim f_a/(m_s M) \sim 1/m$. The dynamics of saxion at $|\phi| > f_a$ is described by the harmonic oscillation form with a frequency m . Parametrically, we have $|\phi| \sim f_a$ and $|\dot{\phi}| \sim m f_a (\sim m_s M)$. Thus we obtain

$$\left[\Delta \rho_\phi^{(\text{dis})} \right]_{t > t_f} \sim \int_{t_f} dt \frac{1}{2} \dot{\phi}^2 \times \frac{b\alpha^2 T^3}{\phi^2} \sim b\alpha^2 T^3 m. \quad (4.6)$$

This is smaller than the energy losses in the regime of $t < t_f$ for $m < m_s$.

From the above estimates, it is found that the saxion energy loss due to the dissipation effect in its one oscillation is given by

$$\left[\Delta \rho_\phi^{(\text{dis})} \right]_{\Delta t = m^{-1}} \sim b\alpha^2 T^3 m_s \log \left(\frac{M}{\phi_0} \right). \quad (4.7)$$

We define the effective dissipation rate averaged over the saxion oscillation period:

$$\Gamma_\phi^{(\text{dis},1)} \equiv \frac{m}{\rho_\phi} \left[\Delta \rho_\phi^{(\text{dis})} \right]_{\Delta t = m^{-1}} = \frac{b'\alpha^2 T^3 m_s}{f_a^2 m}, \quad (4.8)$$

where $b' \equiv b \log(M/\phi_0)$.

Now let us show the evolution of the radiation and saxion energy density at a few Hubble time after the thermal inflation. The radiation energy density evolves as

$$\dot{\rho}_{\text{rad}} + 4H\rho_{\text{rad}} = \Gamma_\phi^{(\text{dis},1)}(T)\rho_\phi. \quad (4.9)$$

If $\Gamma_\phi^{(\text{dis},1)} \gg H_{\text{PT}}$, the significant fraction of the saxion energy density soon goes into radiation and the resulting radiation temperature becomes $T = T_{\text{TI}} \sim \sqrt{m f_a}$. In the opposite limit, after a few Hubble time, the radiation temperature becomes

$$T = \max[1, f] T_{\text{PT}} \equiv F T_{\text{PT}}, \quad (4.10)$$

where

$$f \equiv \left(b'\alpha^2 \lambda \frac{M_{\text{Pl}}}{f_a} \right) \sim 10^9 (b'\alpha^2 \lambda) \left(\frac{10^9 \text{ GeV}}{f_a} \right), \quad (4.11)$$

by noting that $\dot{T} \sim \text{const}$ from (4.9). Therefore, we obtain

$$T = \min [FT_{\text{PT}}, T_{\text{TI}}], \quad (4.12)$$

at the end of first stage after thermal inflation.

By comparing the dissipation rate with the Hubble rate at the end of thermal inflation, we obtain

$$\frac{\Gamma_{\phi}^{(\text{dis},1)}}{H_{\text{PT}}} \sim F^3 b' \alpha^2 \lambda \frac{m_s^4 M_{\text{pl}}}{\lambda^4 m^2 f_a^3} \sim 10^2 \times (10 F^3 b' \alpha^2 \lambda) \left(\frac{m_s/\lambda}{1 \text{ TeV}} \right)^4 \left(\frac{1 \text{ GeV}}{m} \right)^2 \left(\frac{10^9 \text{ GeV}}{f_a} \right)^3. \quad (4.13)$$

The ratio (4.13) can be much larger than unity, meaning that the saxion can dissipate most of its energy within one Hubble time after the thermal inflation ends and the radiation with a temperature $T = T_{\text{TI}}$ is produced. Otherwise, the saxion coherent oscillation soon dominates the Universe and it decays into the axion pair, as we will see below, leading to unsuccessful reheating. Therefore we must have $\Gamma_{\phi}^{(\text{dis},1)}/H_{\text{PT}} \gg 1$.

However, even if $\Gamma^{(\text{dis},1)}$ is initially much larger than H_{PT} , all the energy of the saxion oscillation is not dissipated away; this is because the dissipation rate is significantly suppressed once the smallest value of ϕ during one oscillation becomes larger than M . Thus, the coherent oscillation around $|\phi| = f_a$, whose energy density is the same order of the initial total saxion energy density ($\rho_{\phi} \sim m_s^2 M^2 \sim m^2 f_a^2$), still remains after such a dissipation. Then we need to consider the second stage of saxion reheating.

4.2.2 Second stage

At the second stage, the saxion oscillates around the minimum $|\phi| = f_a$. Here we assume that the condition $\Gamma_{\phi}^{(\text{dis},1)} \gg H_{\text{PT}}$ is satisfied and hence $T = T_{\text{TI}}$. The thermal dissipation rate at this epoch is given by

$$\Gamma_{\phi}^{(\text{dis},2)} = \frac{b \alpha^2 T^3}{f_a^2}. \quad (4.14)$$

By comparing it with the Hubble rate at the end of thermal inflation, we obtain

$$\frac{\Gamma_{\phi}^{(\text{dis},2)}}{H_{\text{PT}}} \sim \frac{b \alpha^2 m^{1/2} M_{\text{pl}}}{f_a^{3/2}} \sim 10^3 b \left(\frac{\alpha}{0.1} \right)^2 \left(\frac{m}{1 \text{ GeV}} \right)^{1/2} \left(\frac{10^9 \text{ GeV}}{f_a} \right)^{3/2}. \quad (4.15)$$

Here we have substituted $T = T_{\text{TI}}$.

At the second stage, as well as the dissipation process due to the thermal bath, there is another important process which reduces the energy density of the saxion oscillation. In the present analysis, we consider the case where the saxion dominantly decays into the axion pair. Then, the decay rate of the saxion is estimated as

$$\Gamma_{\phi \rightarrow 2a} = \frac{1}{64\pi} \frac{m^3}{f_a^2}. \quad (4.16)$$

The decay must happen before the saxion again comes to dominate the Universe for the successful thermal history. Note that the ratio between $\Gamma_{\phi \rightarrow 2a}$ and $\Gamma_{\phi}^{(\text{dis},2)}$ is given by

$$\frac{\Gamma_{\phi}^{(\text{dis},2)}(T_{\text{TI}})}{\Gamma_{\phi \rightarrow 2a}} = 64\pi b\alpha^2 \left(\frac{f_a}{m}\right)^{3/2}, \quad (4.17)$$

and it is much larger than unity, hence the decay into axion is negligible just at the end of thermal inflation. However, as the temperature decreases, the decay rate into the axion pair becomes dominant. In particular, even if $\Gamma_{\phi}^{(\text{dis},2)}$ becomes larger than the expansion rate just after the thermal inflation, the saxion coherent oscillation cannot be fully dissipated away because the dissipation rate becomes suppressed with the decrease of the cosmic temperature. (See Appendix.) Thus, the decay of the saxion may overproduce the axion. If the saxion still does not dominate the Universe when the perturbative decay into the axion becomes efficient ($H \sim \Gamma_{\phi \rightarrow 2a}$), the axion overproduction is avoided.

In order to estimate the present energy density of axion, we have performed numerical calculation. We have solved a set of equations

$$\dot{\rho}_{\phi} + (3H + \Gamma_{\phi}^{(\text{dis})} + \Gamma_{\phi \rightarrow 2a})\rho_{\phi} = 0, \quad (4.18)$$

$$\dot{\rho}_{\text{rad}} + 4H\rho_{\text{rad}} = \Gamma_{\phi}^{(\text{dis})}\rho_{\phi} + \Gamma_a^{(\text{dis})}\rho_a, \quad (4.19)$$

$$\dot{\rho}_a + 4H\rho_a = \Gamma_{\phi \rightarrow 2a}\rho_{\phi} - \Gamma_a^{(\text{dis})}\rho_a, \quad (4.20)$$

$$3H^2 M_{\text{pl}}^2 = \rho_{\phi} + \rho_{\text{rad}} + \rho_a. \quad (4.21)$$

with initial conditions $T = T_{\text{PT}}$, $\rho_a = 0$ and $\rho_{\phi} = m^2 f_a^2$, where $\Gamma_a^{(\text{dis})}$ denotes the axion dissipation rate, which is roughly same as $\Gamma_{\phi}^{(\text{dis})}$. Then we have calculated the extra effective number of neutrino species ΔN_{eff} by

$$\Delta N_{\text{eff}} = \frac{43}{7} \left(\frac{10.75}{g_{*s}}\right)^{1/3} \left(\frac{\rho_a}{\rho_{\text{rad}}}\right), \quad (4.22)$$

where g_{*s} is the effective number of massless degrees of freedom. The results are plotted in Figs. 2. In this plot we have taken $f_a/M = 100$ (hence $m_s \sim 100m$) and $\lambda = 1$ (top) and $\lambda = 0.05$ (bottom), and $g_{*s} = 228.75$. The recent Planck result [46] excludes the parameter regions with $\Delta N_{\text{eff}} \gtrsim 1$. It is seen that parameters with $m \gtrsim$ a few TeV and $f_a \gtrsim 10^9$ GeV is viable. ($f_a \gtrsim 4 \times 10^8$ GeV is required from astrophysical arguments [22].) Therefore, heavy SUSY scenario with $m \gg O(1)$ TeV is favored.^{#14} We have checked that the figure looks quite similar for $f_a/M = 1$. To see more detail, we have plotted time evolution of various quantities as a function of H_{PT}/H in Fig. 3. Parameters are chosen as $\lambda = 1$, $m = 5$ TeV, $f_a = 10^9$ GeV, $M = 10^{-2}f_a$ with which we obtain $\Delta N_{\text{eff}} \sim 1$. Since the dissipation rate $\Gamma_{\phi}^{(\text{dis})}$ is larger than the Hubble rate at early time, a significant amount of

^{#14} Thermal inflation occurs for most of the parameter regions in the top panel of Fig. 2. In the bottom panel, thermal inflation does not take place in the about upper half of the parameter region (see Eq. (4.3)).

the saxion energy goes to the radiation. The produced axions are also thermalized since $\Gamma_a^{(\text{dis})}$ is also large. Although gradually $\Gamma_\phi^{(\text{dis})}/H$ decreases, the perturbative decay $\Gamma_{\phi \rightarrow 2a}$ becomes efficient before the saxion dominates the Universe and hence the overproduction of axion can be avoided.

Note that in a case with large dissipation rate $\Gamma_\phi^{(\text{dis})} \gg H$, axinos and saxions will be thermally populated. This does not cause cosmological problems as long as the axino is heavy enough to decay well before BBN and the produced LSPs have relatively large annihilation cross section [47, 48, 49]. All these necessary properties are consistent with heavy SUSY scenario, such as the pure gravity-mediation [50], which leads to 125 GeV Higgs [1].

5 Conclusions and Discussion

In this paper, we have investigated the dynamics of a scalar field in thermal environment with a symmetry breaking potential. In particular, we have performed a detailed study of the trapping of scalar field at an enhanced symmetry point.

Throughout this paper, we have focused on the case where the scalar field interacts with fermion and boson which are charged under SM gauge group and the initial amplitude of scalar field is far from the minimum of the potential. We considered the trapping dynamics of scalar field with the interaction of SM plasma taken into account. Although we have studied the SUSY PQ model in detail to make our discussion concrete, we emphasize that our results are rather general and that they are applicable to other symmetry breaking dynamics than the saxion. Since the trapping occurs due to the particle production, there are two possible sources: the production from the background thermal environment or the non-perturbative production from saxion itself. It is also notable that the higher order correction to thermal potential becomes important, so-called thermal logarithmic potential. We have taken into account all these effects carefully and found that the dynamics of scalar field complicatedly depends on the interactions of produced particles to the SM particles. However, even if the saxion has a large initial amplitude, it is found that the saxion is likely trapped at its origin once and often leads to the thermal inflation for the most parameters of our interest.

We have also studied the dynamics of saxion after the phase transition. It is noticeable that, even if the saxion once dominates the Universe and the thermal inflation occurs, the saxion can successfully dissipate its energy by the interaction with the thermal bath. In such a case, we can avoid the overproduction of the axion by the decay of the saxion field even if the dominant decay mode of the free saxion is into axion pair. To verify this statement, we have performed numerical calculation, and it is shown that the axion overproduction does not occur in the parameter region that are consistent with the high-scale SUSY scenario.

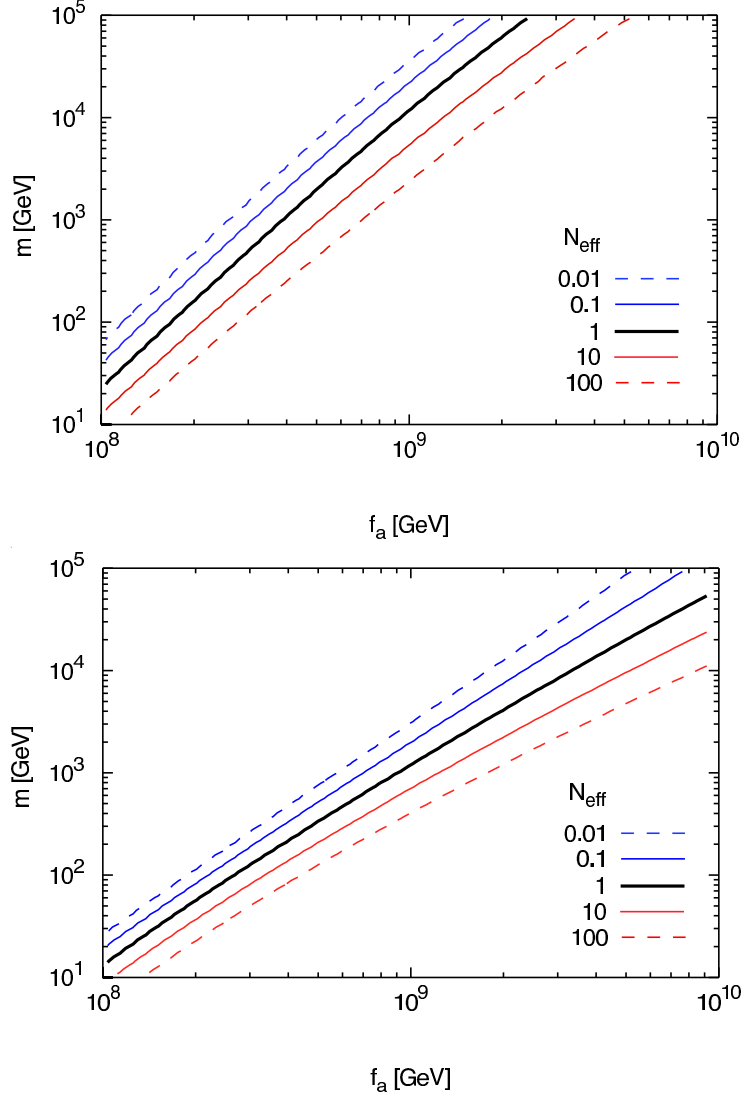


Figure 2: Contours of N_{eff} on (f_a, m) plane. We have taken $f_a/M = 100$ (hence $m_s \sim 100m$) and $\lambda = 1$ (top) and $\lambda = 0.05$ (bottom).

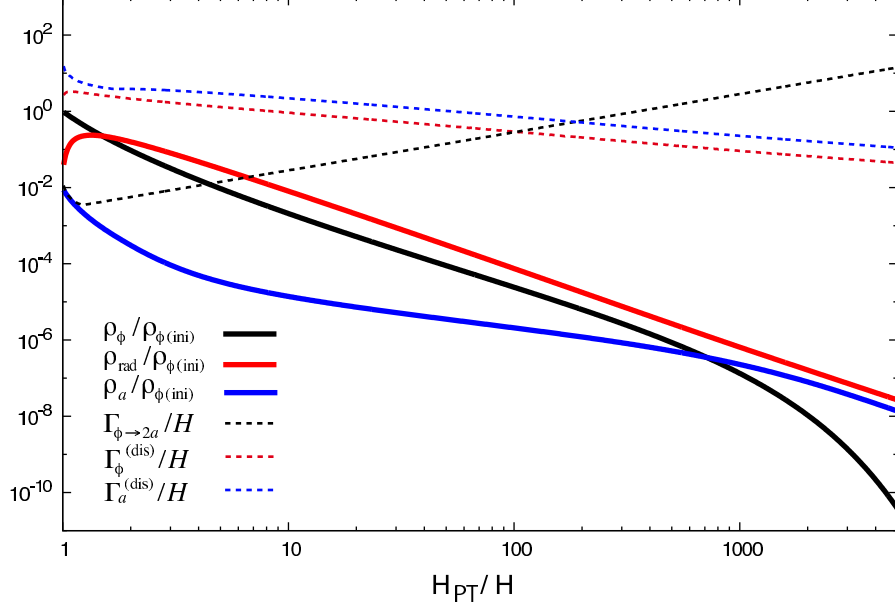


Figure 3: Time evolution of various quantities as a function of H_{PT}/H : ρ_ϕ , ρ_{rad} , ρ_a normalized by the initial saxion energy density $\rho_{\phi(\text{ini})}$, and $\Gamma_{\phi \rightarrow 2a}$, $\Gamma_\phi^{(\text{dis})}$, $\Gamma_a^{(\text{dis})}$ normalized by H . Parameters are chosen as $\lambda = 1$, $m = 5 \text{ TeV}$, $f_a = 10^9 \text{ GeV}$, $M = 10^{-2} f_a$.

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Appendix

A Time Evolution of ρ_ϕ

Let us consider the evolution of the ρ_ϕ in the presence of time-dependent dissipation effect $\Gamma(t)$:

$$\dot{\rho}_\phi(t) + \Gamma(t)\rho_\phi(t) = 0. \quad (\text{A.1})$$

Generally, $\Gamma(t)$ is parametrized as $\Gamma(t) = \Gamma_i(t_i/t)^n$ where t_i is an initial time. In the case of standard perturbative decay, $n = 0$. The case of $n = 1$ includes the Hubble friction: $\Gamma(t) = \Gamma_i t_i/t$ with $\Gamma_i t_i = 1/2(2/3)$ corresponding to the RD (MD) Universe. Thermal dissipation effect may correspond to $n > 1$. This equation can be easily integrated to

obtain

$$\frac{\rho_\phi(t)}{\rho_\phi(t_i)} = \begin{cases} \left(\frac{t_i}{t}\right)^{\Gamma_i t_i} & \text{for } n = 1, \\ \exp\left[\frac{\Gamma_i t_i}{n-1} \left\{\left(\frac{t_i}{t}\right)^{n-1} - 1\right\}\right] & \text{for } n \neq 1. \end{cases} \quad (\text{A.2})$$

From this expression it is easily checked that ρ_ϕ exponentially decays with time for $n < 1$. For $n > 1$, the energy density exponentially decreases at $t \gtrsim (\text{a few}) \times t_i$ if $\Gamma_i t_i \gg 1$. Therefore, if $\Gamma_i t_i \gg 1$, ϕ loses most of its energy within a few Hubble time. Eventually ρ_ϕ approaches to the asymptotic value

$$\rho_\phi(t) \rightarrow \rho_\phi(t_i) \exp\left[-\frac{\Gamma_i t_i}{n-1}\right], \quad (\text{A.3})$$

at $t > t_f$ where t_f is defined by $t_f \Gamma(t_f) = 1$.

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